Introductory Linear Algebra-Complex Numbers Home Assignment

MATH 250 (Instructor: Tom Benhamou) Oct 35, 2024

Problem 1. Simplify the following expressions to the form $a + ib$:

(1) $(-1 - i)(3 + 2i) + (1 + i)$.

(2) $(-5 + 3i)^2$.

(3) $\frac{17+5i}{2+i}$

(4) $\frac{2+3i}{5-2i}$

Problem 2. Solve the following complex equation:

$$(2i + 3)z + 3i = 5 - 3i$$

Problem 3. Find all solutions to the following system of equations (you can use complex matrix elimination):

$$\begin{cases} 2z_1 + (2i + 1)z_2 = 1 \\ (2i - 1)z_1 + (3i - 1)z_2 = 2i \end{cases}$$

Problem 4. Consider the vectors

$$\bar{u}_1 = \begin{bmatrix} 1 + i \\ 1 - i \\ 2 \end{bmatrix}, \bar{u}_2 = \begin{bmatrix} 3i \\ 1 - i \\ 1 + i \end{bmatrix}, \bar{u}_3 = \begin{bmatrix} 2i + 1 \\ 0 \\ 1 \end{bmatrix}$$

in $\mathbb{C}^3$. Are the vectors $\bar{u}_1, \bar{u}_2, \bar{u}_3$ linearly independent? namely, are there scalars $z_1, z_2, z_3 \in \mathbb{C}$, not all of them are zero, such that

$$z_1\bar{u}_1 + z_2\bar{u}_2 + z_3\bar{u}_3 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$